

1 Problem:

Be M of the Form $M = \begin{pmatrix} A & B \\ 0 & C \end{pmatrix}$, where $M \in \mathbb{K}^{(p+q) \times (p+q)}$, $A \in \mathbb{K}^{p \times p}$, $C \in \mathbb{K}^{q \times q}$, $B \in \mathbb{K}^{p \times q}$, 0 denotes the 0-Matrix from $\mathbb{K}^{q \times p}$. Prove that $\det(M) = \det(A) * \det(C)$

2 Solution:

By replacing rows with the sum of those rows and multiples of other rows, M can be brought into lower triangle form:

$$M' = \begin{pmatrix} A' & B' \\ 0 & C' \end{pmatrix}.$$

where A' , B' are of the form

$$Z = \begin{pmatrix} z_{11} & \cdot & \cdot & \cdot & \cdot & z_{1r} \\ 0 & z_{12} & \cdot & \cdot & \cdot & z_{2r} \\ \cdot & & \cdot & & & \cdot \\ \cdot & & & \cdot & & \cdot \\ \cdot & & & & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & 0 & z_{rr} \end{pmatrix}$$

So, A and C are also brought into lower triangle form when doing this, and $\det(M) = \det(M')$ (Because addition of multiples of rows of M to other rows does not change the determinant), $\det(A) = \det(A')$, $\det(C) = \det(C')$ (Because only multiples of rows from A (C) are added to rows from A (C), since M has the 0-Matrix in the lower left sector).

Therefore:

$$\det(A) = a_{11} * a_{22} * \dots * a_{pp}$$

$$\det(C) = c_{11} * c_{22} * \dots * c_{qq}$$

$$\det(M) = a_{11} * a_{22} * \dots * a_{pp} * c_{11} * c_{22} * \dots * c_{qq} = \det(A) * \det(C)$$

□